

Optimization of rhombic drive mechanism used in beta-type Stirling engine based on dimensionless analysis



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ABSTRACT

In the present study, optimization of rhombic drive mechanism used in a beta-type Stirling engine is performed based on a dimensionless theoretical model toward maximization of shaft work output. Displacements of the piston and the displacer with the rhombic drive mechanism and variations of volumes and pressure in the chambers of the engine are firstly expressed in dimensionless form. Secondly, Schmidt analysis is incorporated with Senft's shaft work theory to build a dimensionless thermodynamic model, which is employed to yield the dimensionless shaft work. The dimensionless model is verified with experimental data. It is found that the relative error between the experimental and the theoretical data in dimensionless shaft work is lower than 5.2%. This model is also employed to investigate the effects of the influential geometric parameters on the shaft work, and the optimization of these parameters is attempted. Eventually, design charts that help design the optimal geometry of the rhombic drive mechanism are presented in this report.

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1. Introduction

Stirling engines are potentially competitive when applied in a number of emerging engineering technologies since the engines are referred to as the external-combustion engines which are suitable for a variety of external heat sources. For example, the engines using nuclear energy could power deep-space exploration probes [1], using fossil or hydrogen fuel could drive a vehicle like an automobile [2], or a submarine [3], and using a solar dish collector could produce electricity [4]. A typical solar dish apparatus consists of a large dish aimed at the sun to reflect the rays into the focus point, which is located at a thermal receiver of the Stirling engine. The engine operates by converting the heat received from the solar dish into mechanical work. The work output of the Stirling cycle is then used to drive a generator and create electricity. Furthermore, the electrical power generated by the Stirling engine can be further used to electrolyze water for producing hydrogen. The hydrogen gas could be stored properly and then fed into a fuel cell for electrical power generation when needed [5].

As reviewed by Senft [6], the Stirling engine was named after its inventor Robert Stirling who invented his first engine in 1816. The first successful theoretical analysis was presented by Schmidt [7] in 1871. Schmidt's analysis led to a simple closed-form solution for

indicated work of the engine. Lately, a large numbers of Stirling engine prototypes with various configurations, which have been categorized in to α -, β -, and γ -type Stirling engines, have been created [8]. In addition, some single-piston designs, such as thermal-lag Stirling engine [9] and pulse tube engine [10], are also presented. On the other hand, development in thermodynamic models [11–14], optimization methods [15–18], and measurement methods for engine performance [19–21] is also advanced by the progress in prototype building. Based on Schmidt's model, a group of researchers, such as Finkelstein [15], Kirkley [16] and Walker [17] investigated the optimal phase angle and swept volume ratio for various types of Stirling machines. Senft [13] introduced a concept of effectiveness of mechanism and proposed a relation between the forced work and the shaft work of the engine. Later, the same author [18] combined the Schmidt theory and the shaft work theory to carry out the optimal combination of the phase angle and the swept volume ratio for the γ -type Stirling engine. Recently, theoretical analysis was extended to the irreversible cycle analysis. Cheng and Yu [12] proposed a thermodynamic model for the β -type Stirling engine with rhombic-drive mechanism by taking into account the effectiveness of the regenerative channel as well as the thermal resistances of the heating and cooling heads. The thermodynamic model was then incorporated with a dynamic model to perform a dynamic simulation model for the β -type Stirling engines with cam-drive mechanisms by the same group of authors [22].

According to studies of and Kirkley [16] and Cheng and Yang [23], it is recognized that among α -, β -, and γ -type Stirling engines,

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Nomenclature

A	cross section area (m^2)
e	l/r
E	effectiveness of mechanism
l	length of linkage (m)
m	mole number of working fluid (mol)
p, p_b	pressure and buffer pressure (Pa)
r	offset distance from the crank to the center of shaft (m)
R	gas constant $R = 8.314$ (J/K mol)
R	radius of gear (m)
S	stroke (m)
T	temperature (K)
V	volume (m^3)
W_i, W_f, W_s	dimensionless indicated work, forced work, and shaft work
y_0, y_1	lengths of displacer and piston rods (m)
y_d, y_p	locations of top and bottom links of rhomboid (m)

Greek symbols

α_d	displacement phase angle (deg)
α_v	volume phase angle (deg)
γ	compression ratio

ε	$(R - l)/r$
θ	crank angle (deg)
κ	swept volume ratio (V_{sc}/V_{se})
τ	temperature ratio (T_c/T_e)
χ	total dead volume ratio

Superscripts

–	dimensionless quantity
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Subscripts

c	compression chamber
d	displacer
e	expansion chamber
h	heater
k	cooler
l	lower yoke
max, min	maximum and minimum
p	piston
r	regenerator
s	swept volume
t	total
u	upper yoke

the β type possesses higher power density than the other two types. Therefore, the attention of this study is focused on the beta-type Stirling engine. Furthermore, a mechanism is typically installed with the beta-type Stirling engine to drive a displacer and a piston in order to fulfill the required phase angle between the two moving parts as well as the designed strokes of the displacer and the piston. The drive mechanism with the beta-type Stirling engine could be a mechanism of crank drive, bell-crank drive, twin inclined Scotch yoke drive, spring drive, or rhombic drive [6,24–26]. As was

described in Ref. [26], the rhombic drive uses a jointed rhomboid to convert linear motion of the reciprocating piston and displacer to a rotation of the flywheel. In the rhombic drive mechanism, one rigid rod is installed connecting the piston to the top link of the jointed rhomboid, and another rigid rod connecting the displacer to the bottom link of the rhomboid. Two symmetric gears of equal diameter are connected to the right and the left links of the rhomboid fixed on the gears at an offset distance from respective gear centers. The two gears are in contact and rotate in opposite

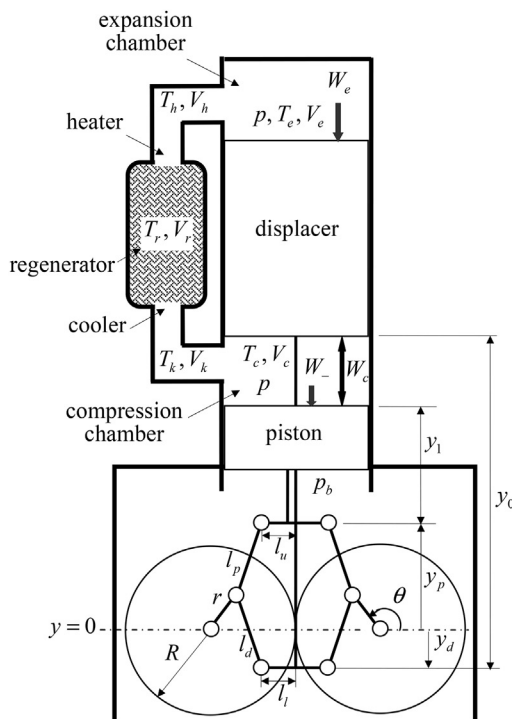


Fig. 1. Schematic and photo of 300-W beta-type Stirling engine with rhombic-drive mechanism [Model NCKU300-2, developed by Power Engines and Clean Energy Laboratory (PEACE Lab.), National Cheng Kung University].

directions. The rhombic drive mechanism is a symmetric mechanism that makes the co-axial motion of the displacer and the piston move smoothly to minimize the lateral force acting perpendicular to the axis, and hence, the friction at contact surface between the piston and the cylinder can be greatly reduced. For this reason, the rhombic drive mechanism is frequently adopted by the β -type Stirling engine designers.

More recently, the present authors built a 300-W β -type Stirling engine with a rhombic drive mechanism (Model NCKU300-2), of which the photograph is shown in Fig. 1. According to the experiences learned from the development of the engine, it is recognized that major fundamental problems that the designers of the Stirling engines encountered are the time-consuming design process as well as the costly manufacturing procedure. In most of the cases the determination of the geometric parameters of the rhombic drive is based on the experience of the designers or on insufficient experimental information. This is why the engine design can only be carried out in a relatively high-cost time-consuming trial-and-error process. Unfortunately, there are no desired explicit guidelines for design of the geometry of the rhombic drive mechanism in the existing reports.

Under these circumstances, optimization of the rhombic drive mechanism used in the β -type Stirling engine is attempted in this study based on a dimensionless theoretical model. Schmidt analysis is incorporated with Senft's shaft work theory [13,18] to build the dimensionless thermodynamic model, which is capable of providing the dimensionless shaft work under various combinations of the influential geometric parameters. The dimensionless thermodynamic model is verified with experimental data, and the optimal geometric parameters are so determined that the shaft work can be maximized. Eventually, the design charts that can be used as the guidelines for design of the geometry of the rhombic drive mechanism are provided.

2. Dimensionless analytical model

2.1. Displacements of piston and displacer

In the modeling, it is assumed that (1) the transient variations in the volumes of the working spaces inside the engine are not sinusoidal functions of time, but determined from the geometric relations among the parts of the mechanism, (2) the temperatures in the working spaces are unequal but constant, and (3) the pressures in the working spaces are varying with time periodically but are uniform at any time instant.

The dimension variables and the schematic of the β -type Stirling engine with the rhombic drive mechanism are shown in Fig. 1. According to the relations among the variables with the rhombic drive mechanism, the heights of top link of the rhomboid connected to the piston ($\bar{y}_p$) and bottom link of the rhomboid connected to the displacer ($\bar{y}_d$) can be expressed in dimensionless form, respectively, as

$$\bar{y}_p(\theta) = \sin \theta + \sqrt{e_p^2 - (\varepsilon_p + \cos \theta)^2} \quad (1a)$$

$$\bar{y}_d(\theta) = \sin \theta - \sqrt{e_d^2 - (\varepsilon_d + \cos \theta)^2} \quad (1b)$$

where the dimensionless parameters are defined as below:

$$\bar{y}_p = \frac{y_p}{r}, \bar{y}_d = \frac{y_d}{r}, e_p = \frac{l_p}{r}, e_d = \frac{l_d}{r}, \varepsilon_p = \frac{R - l_u}{r}, \varepsilon_d = \frac{R - l_l}{r} \quad (2a-f)$$

It is noted that in practices the relations among these above dimensionless parameters must meet the following criteria

$$e_p - 1 > \varepsilon_p \geq 1 \text{ and } e_d - 1 > \varepsilon_d \geq 1 \quad (3)$$

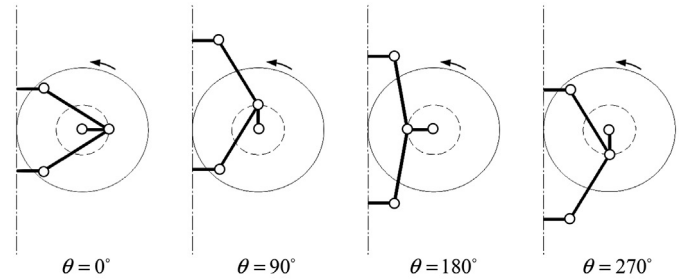
Fig. 2(a) shows the snapshots of the typical rhombic drive mechanism in a cycle, which meets the above valid ranges of the parameters. However, if one of the criteria is not obeyed, a collision between the parts of the mechanism may occur. As firstly shown in Fig. 2(b), for the cases with $\varepsilon_p < 1$ ($R - l_u$ shorter than r) or $\varepsilon_d < 1$ ($R - l_l$ shorter than r), joint A is very possible to collide with the displacer rod at a high rotation speed. In Fig. 2(b), it is at $\theta = 180^\circ$ that the collision may take place. Secondly, as $e_p - 1 < \varepsilon_p$ ($l_p - r$ smaller than $R - l_u$) or $e_d - 1 < \varepsilon_d$ ($l_d - r$ smaller than $R - l_l$), the mechanism is not able to complete one cycle due to interference among the links (see $\theta = 0^\circ$).

Based on Eqs. (1a) and (1b), the maximum and minimum heights of the top and the bottom links can be derived. The results are

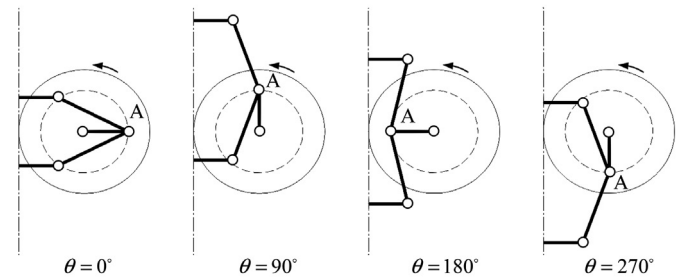
$$\bar{y}_{p,\max} = \sqrt{(e_p + 1)^2 - \varepsilon_p^2} \quad (4a)$$

$$\bar{y}_{p,\min} = \sqrt{(e_p - 1)^2 - \varepsilon_p^2} \quad (4b)$$

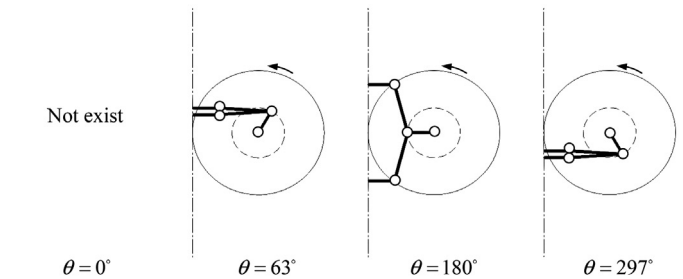
$$\bar{y}_{d,\max} = -\sqrt{(e_d - 1)^2 - \varepsilon_d^2} \quad (4c)$$



(a) $e_p - 1 > \varepsilon_p \geq 1$ and $e_d - 1 > \varepsilon_d \geq 1$ (valid)



(b) $\varepsilon_p < 1$ or $\varepsilon_d < 1$ (invalid)



(c) $e_p - 1 < \varepsilon_p$ or $e_d - 1 < \varepsilon_d$ (invalid)

Fig. 2. Valid and invalid parameter ranges. (a) $e_p - 1 > \varepsilon_p \geq 1$ and $e_d - 1 > \varepsilon_d \geq 1$ (valid) (b) $\varepsilon_p < 1$ or $\varepsilon_d < 1$ (invalid) (c) $e_p - 1 < \varepsilon_p$ or $e_d - 1 < \varepsilon_d$ (invalid).

$$\bar{y}_{d,\min} = -\sqrt{(e_d + 1)^2 - \varepsilon_d^2} \quad (4d)$$

The dimensionless strokes with the piston and the displacer can be yielded as

$$\bar{S}_p = \bar{y}_{p,\max} - \bar{y}_{p,\min} \quad (5)$$

$$\bar{S}_d = \bar{y}_{d,\max} - \bar{y}_{d,\min} \quad (6)$$

where $\bar{S}_p = S_p/r$ and $\bar{S}_d = S_d/r$. Thus, one obtains the relation for the swept volume ratio

$$\kappa = \frac{\bar{S}_p}{\bar{S}_d} = \frac{\sqrt{(e_p + 1)^2 - \varepsilon_p^2} - \sqrt{(e_p - 1)^2 - \varepsilon_p^2}}{\sqrt{(e_d + 1)^2 - \varepsilon_d^2} - \sqrt{(e_d - 1)^2 - \varepsilon_d^2}} \quad (7)$$

The displacer leads the piston by a phase angle, which is readily determined by the difference in angle between $\bar{y}_{d,\max}$ and $\bar{y}_{p,\max}$ as

$$\alpha_d = (180/\pi) \left[\pi - \cos^{-1} \left(\frac{\varepsilon_p}{e_p + 1} \right) - \cos^{-1} \left(\frac{\varepsilon_d}{e_d - 1} \right) \right] \quad (\text{in deg}) \quad (8)$$

Note that the height of the bottom surface of the displacer can be determined by $\bar{y}_d + \bar{y}_0$, and that of the top surface of the piston by $\bar{y}_p + \bar{y}_1$, where $\bar{y}_0$ and $\bar{y}_1$ are the dimensionless lengths of the displacer rod and the piston rod, respectively. The mechanism should be properly design to minimize the dead volume, and hence the minimum difference in a cycle between $\bar{y}_d(\theta) + \bar{y}_0$ and $\bar{y}_p(\theta) + \bar{y}_1$ is set to be zero such that the relation between $\bar{y}_0$ and $\bar{y}_1$ must satisfy the following equation

$$\bar{y}_0 - \bar{y}_1 = \sqrt{(e_p + e_d)^2 - (\varepsilon_p - \varepsilon_d)^2} \quad (9)$$

Fig. 3 shows the dimensionless displacements of the bottom surface of displacer and the top surface of piston at $e_p = e_d = 2.5$ and $\varepsilon_p = \varepsilon_d = 1$. The crank angle at which the minimum difference in a cycle between $\bar{y}_d(\theta) + \bar{y}_0$ and $\bar{y}_p(\theta) + \bar{y}_1$ approaches zero is 180° or 540° in this figure. It is also noted that the displacement phase angle between the displacer and the piston α_d is approximately 58.412° , which is obtained by using Eq. (8).

2.2. Volume variations of working spaces

The volumes in the expansion space and the compression space caused by the movement of the piston and the displacer in an ideal situation with zero dead volume in the expansion space and the compression space can be written as

$$\begin{aligned} V_e(\theta) &= A_d r [\bar{y}_{d,\max} - \bar{y}_d(\theta)] \\ V_c(\theta) &= A_p r \{ [\bar{y}_d(\theta) + \bar{y}_0] - [\bar{y}_p(\theta) + \bar{y}_1] \} \end{aligned} \quad (10)$$

where A_d and A_p are the cross section areas of the displacer and the piston, respectively. To simplify the model, since the cross-sectional area of the rod connected to the displacer is much less than that of the displacer (about 0.5 ~ 2%), one may ignore the cross-sectional area of the rod and let $A_p \approx A_d$. Also, let V_{se} be the swept volume by the displacer ($= A_d r \bar{S}_d$). One can derive the dimensionless volume variations in the expansion space and the compression space with zero dead volume as

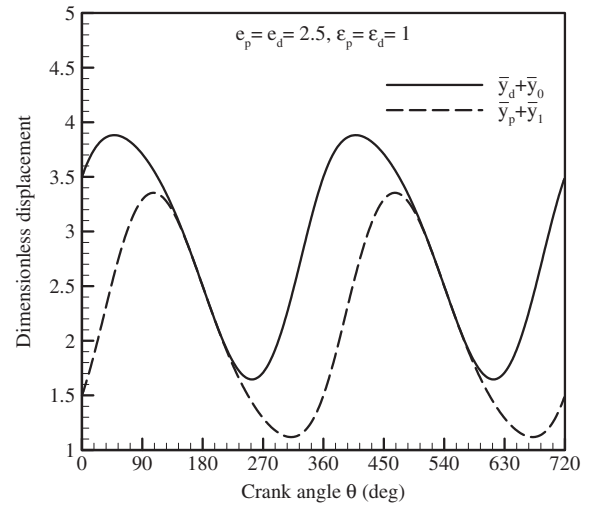


Fig. 3. Dimensionless displacements of top surface of piston and bottom surface of displacer, at $e_p = e_d = 2.5$ and $\varepsilon_p = \varepsilon_d = 1$.

$$\begin{aligned} \bar{V}_e(\theta) &= [\bar{y}_{d,\max} - \bar{y}_d(\theta)] / \bar{S}_d \\ \bar{V}_c(\theta) &= \{ [\bar{y}_d(\theta) + \bar{y}_0] - [\bar{y}_p(\theta) + \bar{y}_1] \} / \bar{S}_d \end{aligned} \quad (11)$$

where $\bar{V}_e(\theta) = V_e/V_{se}$ and $\bar{V}_c(\theta) = V_c/V_{se}$.

Then, dimensionless total volume of all working spaces in the engine can be written as

$$\bar{V}_t(\theta) = \bar{V}_h + \bar{V}_r + \bar{V}_k + \{ (\bar{y}_{d,\max} + \bar{y}_0) - [\bar{y}_p(\theta) + \bar{y}_1] \} / \bar{S}_d \quad (12)$$

The first three terms represent the dimensionless dead volume in the heater, regenerator, and cooler, respectively. Note that the total volume variation is caused by the motion of the piston only. The ratio of the maximum to the minimum values of the total volume $\bar{V}_t(\theta)$ denotes the compression ratio (γ). Introducing a total dead volume ratio $\chi = \bar{V}_h + \bar{V}_r + \bar{V}_k$ to Eq. (12) yields

$$\gamma = 1 + \kappa \bar{S}_d / [\chi \bar{S}_d + (\bar{y}_{d,\max} + \bar{y}_0) - (\bar{y}_{p,\max} + \bar{y}_1)] \quad (13)$$

2.3. Pressure and buffer pressure

In this study, Schmidt theory is employed to determine the pressure in the working spaces as

$$p(\theta) = \frac{mR}{\frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_k}{T_k} + \frac{V_e(\theta)}{T_e} + \frac{V_c(\theta)}{T_c}} \quad (14)$$

where T_e and T_c denote the temperatures in the expansion space and the compression space, respectively. In addition, T_h , T_r , and T_k are the temperatures in the inevitable dead volumes inside the heater, the regenerator, and the cooler. The temperatures in the dead volumes may be further approximated by the mean value of T_e and T_c . Eq. (14) can then be simplified and written in dimensionless form as

$$\bar{p}(\theta) = p(\theta) / \frac{mRT_e}{V_{se}} = \frac{\tau}{2\tau\chi/(1+\tau) + \tau\bar{V}_e(\theta) + \bar{V}_c(\theta)} \quad (15)$$

where $\tau = T_c/T_e$ is referred to as the temperature ratio and $\bar{p}(\theta)$ is the dimensionless pressure. As described by Cheng and Yang [27], the temperature ratio τ is a function of thermal characteristics of the

heater and the cooler, effectiveness of the regenerator, mass of the working gas, and engine speed.

The dimensionless buffer pressure $\bar{p}_b$ in the crank case (or buffer space) needs to be determined. Assume the volume in the crank case is sufficiently large; therefore, the pressure variation caused by the piston motion can be neglected. Thus, the dimensionless buffer pressure $\bar{p}_b$ can be approximated as a constant. Herein, the dimensionless buffer pressure is given by the geometric mean of $\bar{p}_{\max}$ and $\bar{p}_{\min}$, $\sqrt{\bar{p}_{\max}\bar{p}_{\min}}$.

2.4. Indicated, forced, and shaft works

Dimensionless indicated work, forced work and shaft work are defined as:

$$\bar{W}_i = \frac{W_i}{mRT_e}, \quad \bar{W}_f = \frac{W_f}{mRT_e}, \quad \bar{W}_s = \frac{W_s}{mRT_e} \quad (16)$$

With the help of Eqs. (12) and (15), the $\bar{p} - \bar{V}_t$ diagram for the case with $\tau = 0.3$, $\chi = 0.5$, $e_p = e_d = 2.5$ and $\varepsilon_p = \varepsilon_d = 1$ are obtained and plotted in Fig. 4. The dimensionless buffer pressure $\bar{p}_b$ is indicated in the figure. The area inside the $\bar{p} - \bar{V}_t$ is evaluated by integration $\oint \bar{p} d\bar{V}_t$ to yield the dimensionless indicated work $\bar{W}_i$.

$$\bar{W}_i = \oint \bar{p} d\bar{V}_t = - \oint \frac{\bar{p}}{\bar{S}_d} \frac{d\bar{y}_p}{d\theta} d\theta \quad (17)$$

Forced work is the work loss due to work transmission via the mechanism. In a β -type Stirling engine, forced work only acts on the piston and is only dependent on the motion of the piston, resulting from the pressure difference on the both sides of the piston. Based on the definition given by Senft [28], the dimensionless forced work can be calculated by.

$$\bar{W}_f = \oint \left[-\frac{\bar{p} - \bar{p}_b}{\bar{S}_d} \frac{d\bar{y}_p}{d\theta} d\theta \right]^- \quad (18)$$

where the symbol $[\]^-$ means

$$[a]^- = \begin{cases} -a & \text{as } a \leq 0 \\ 0 & \text{as } a > 0 \end{cases} \quad (19)$$

Finally, as described in Ref. [28], one can determine the dimensionless shaft work by

$$\bar{W}_s = E\bar{W}_i - (1/E - E)\bar{W}_f \quad (20)$$

where E is the effectiveness of the mechanism, depending on individual mechanism design, friction, engine speed, lubrication, and so on. The magnitude of E is usually determined by experiments whose typical value is ranged from 0.7 to 0.9. In this study, the value of E is assigned to be 0.8.

3. Experiment and comparison

In this study, performance measurement of Model NCKU300-2 is conducted, and the experimental results are utilized to partially compare with theoretical results for validation. The basic specifications of Model NCKU300-2 are tabulated in Table 1, and the design parameters of the links are shown in Fig. 5. More detailed information of the experiment apparatus has been described in Ref. [29]. The engine is heated by a 1.21-kW infrared heater, and a K-type thermocouple and a PID controller are equipped with the heater in measuring and monitoring the heating temperature, respectively. A torque sensor and an angular speed transducer are installed in between the engine and a hysteresis brake. As the data of the torque and the angular speed are recorded, shaft power of

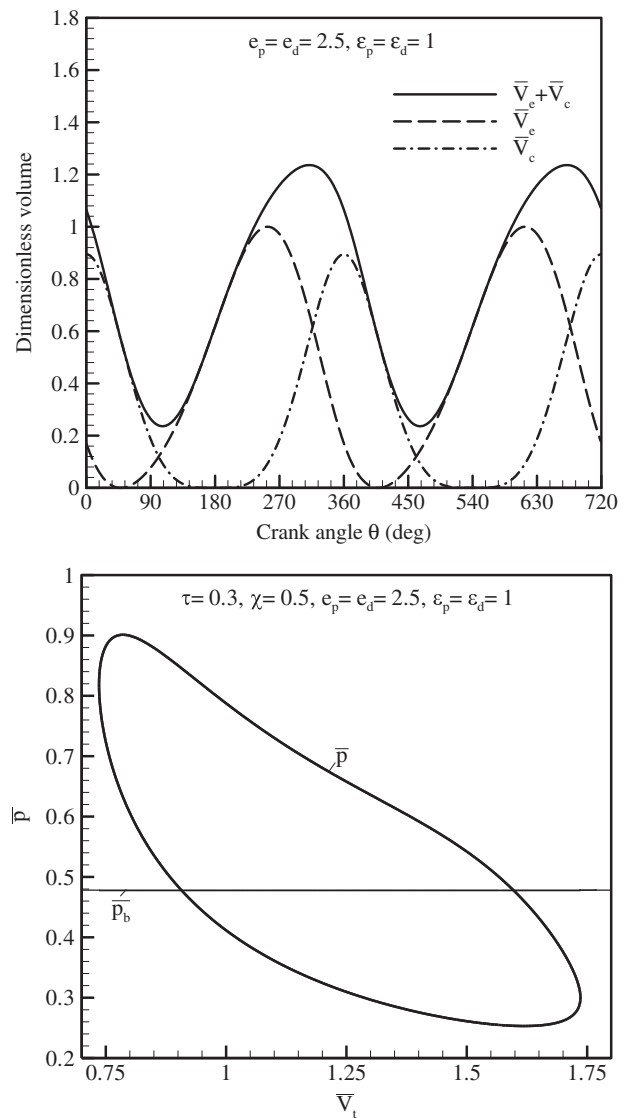


Fig. 4. Dimensionless pressure variations and buffer pressure versus dimensionless total volume variations, at $\tau = 0.3$, $\chi = 0.5$, $e_p = e_d = 2.5$ and $\varepsilon_p = \varepsilon_d = 1$.

Table 1
Specifications of model NCKU300-2.

Parameter	Value
Power capacity	300–500 W
Stroke, S_d	35.219 mm
Cross section area of displacer, A_d	38.485 cm ²
Displacer swept volume, V_{se}	135 c.c.
Displacement phase angle, α_d	54.219
Volume phase angle, α_v	105.537
Swept volume ratio, κ	1
Dead volume ratio, χ	1.478
Compression ratio, γ	1.364
Effectiveness of mechanism, E	0.73
Buffer pressure, p_b	8 atm
Mass of working fluid	0.0752 mol
Working fluid	Helium
Heater type	Tubular heater
Heating temperature	900–1100 K
Cooler type	Fined water jacket
Cooling temperature	300 K (liquid cooling)

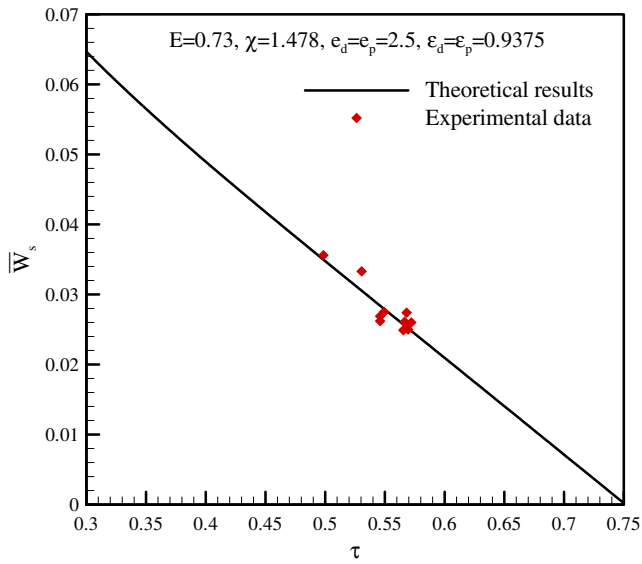


Fig. 5. Comparison between theoretical and experimental results.

the engine can be determined from the product of the torque and the angular speed. As described in Refs. [27,29], at specific heating and cooling temperatures, there exists a maximum shaft power at a certain engine speed. The maximum shaft power and the temperature ratio under different heating temperatures are measured by experiments. Finally, the plot of dimensionless shaft work $\bar{W}_s$ versus temperature ratio τ is yielded and shown in Fig. 5. In this figure, it is found that the temperature ratio is located in the range from 0.49 to 0.58. It is noted that the experimental data agrees with the theoretical results with a relative error lower than 5.2%. The feasibility of the theoretical model is verified; therefore, in the following section a parameter analysis is performed and the geometric optimization of the engine is attempted.

It is noted that the effectiveness of mechanism E can be varied between 0.7 and 0.9 typically. In the pure theoretical analysis section of this study, the value of E is chosen to be the mean value, 0.8 simply for demonstrating a standard parametric analysis. However, as the theoretical model is applied for a specific engine, this value should be determined by fitting the experimental data exclusively.

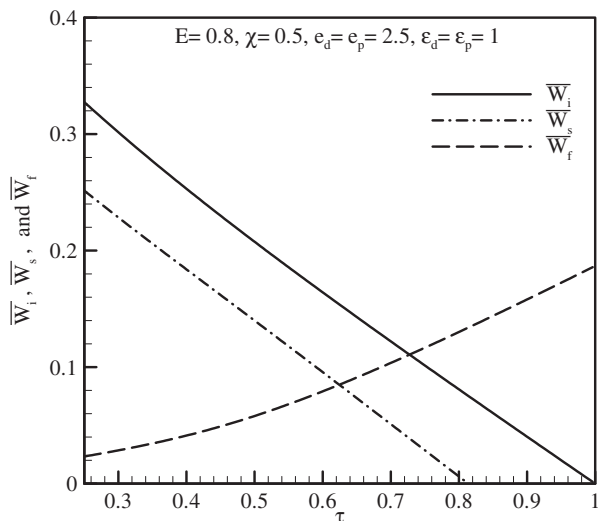


Fig. 6. Effects of temperature ratio on dimensionless works, at $E = 0.8$, $\chi = 0.5$, $e_p = e_d = 2.5$, and $\epsilon_p = \epsilon_d = 1$.

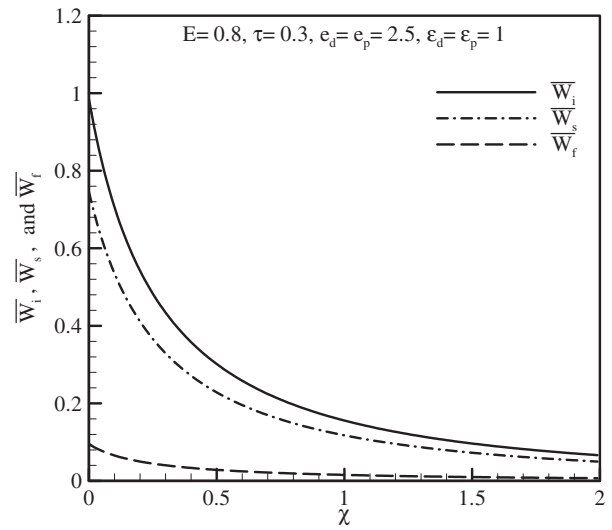
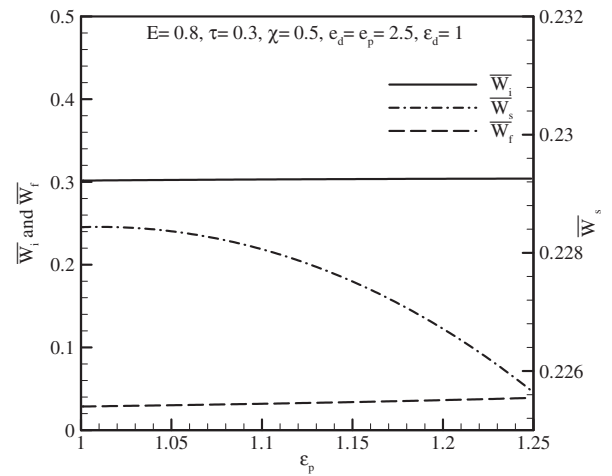
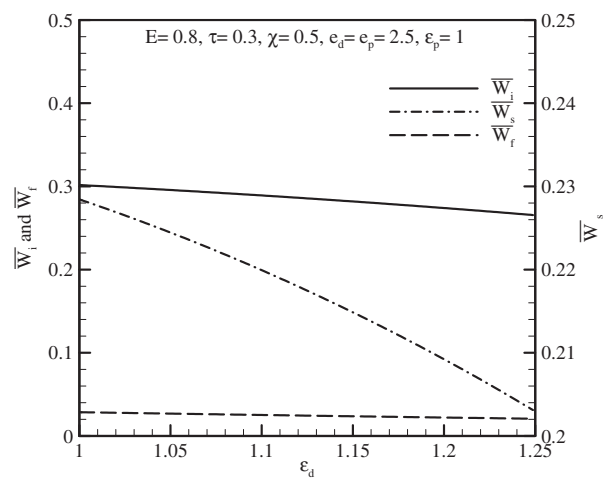


Fig. 7. Effects of total dead volume ratio on dimensionless works, at $E = 0.8$, $\tau = 0.3$, $e_p = e_d = 2.5$, and $\epsilon_p = \epsilon_d = 1$.



(a) Effects of ϵ_p



(b) Effects of ϵ_d

Fig. 8. Effects of ϵ_p and ϵ_d on dimensionless works, at $E = 0.8$, $\tau = 0.3$, $\chi = 0.5$, and $e_p = e_d = 2.5$.

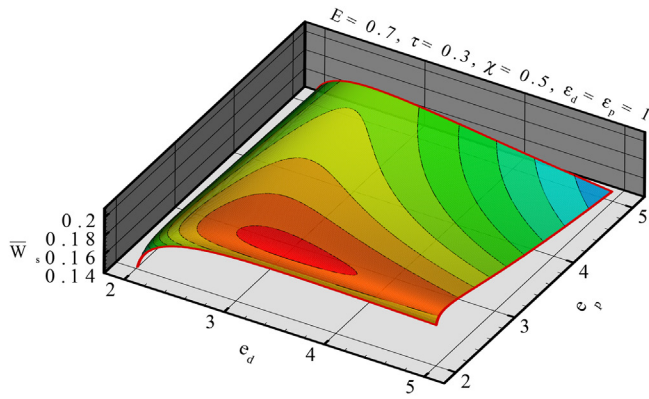
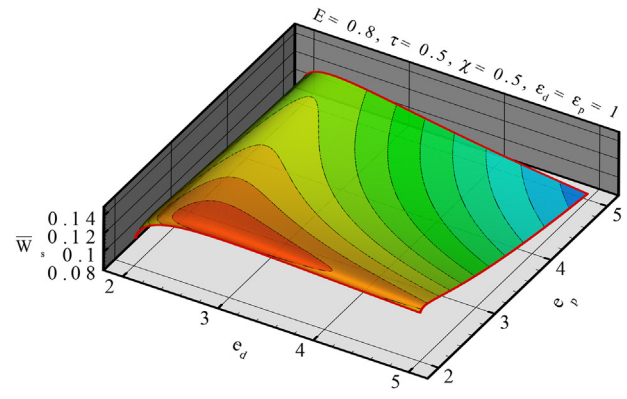
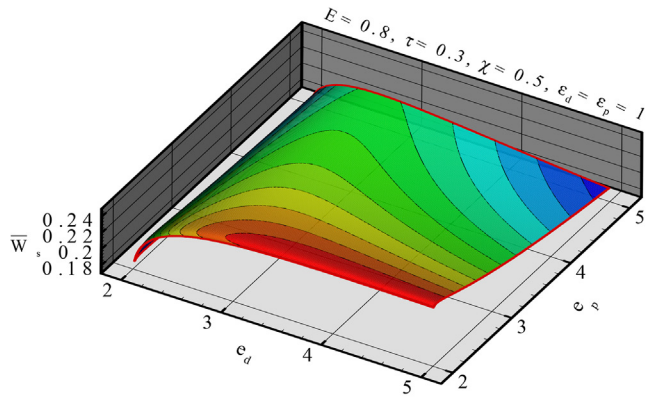
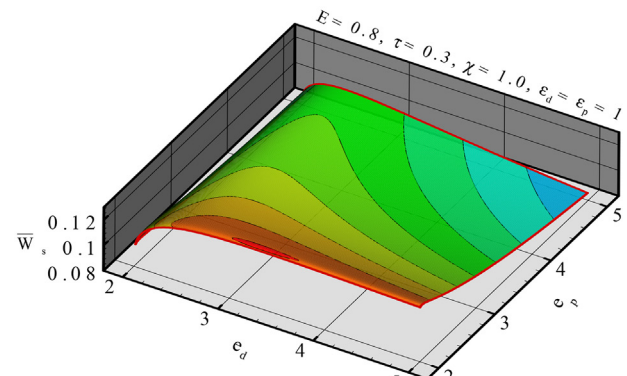
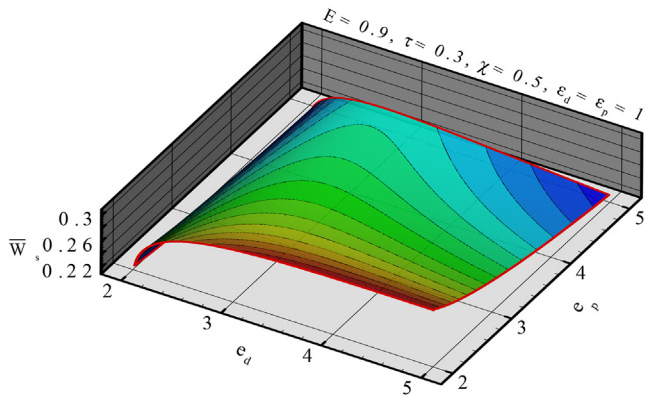
(a) $E = 0.7$ (a) $\tau = 0.5, \chi = 0.5$ (b) $E = 0.8$ (b) $\tau = 0.3, \chi = 1.0$ (c) $E = 0.9$

Fig. 9. Dimensionless shaft work as a function of e_p and e_d for different effectiveness of mechanism at $\tau = 0.3$, $\chi = 0.5$, and $\epsilon_p = \epsilon_d = 1$. (a) $E = 0.7$, (b) $E = 0.8$, (c) $E = 0.9$.

from the performance experiments conducted for the engine. Table 1 shows the specifications of Model NCKU300-2, whose effectiveness of mechanism E is found to be 0.73 in accordance with the experiments.

4. Results and discussion

According to the discussion in the precedent sections, the dimensionless shaft work $\bar{W}_s$ is a function of E , τ , χ , e_p , e_d , ϵ_p , and ϵ_d . As has been described earlier, the magnitude of E is determined by

Fig. 10. Dimensionless shaft work as a function of e_p and e_d for different combinations of τ and χ , at $E = 0.8$ and $\epsilon_p = \epsilon_d = 1$. (a) $\tau = 0.5$, $\chi = 0.5$, (b) $\tau = 0.3$, $\chi = 1.0$.

experiments, whose typical value is ranged from 0.7 to 0.9. Here in this study, the value of E is determined to be 0.8. The present study is aimed to seek optimal combination of these influential parameters toward maximization of the dimensionless shaft work $\bar{W}_s$.

Fig. 6 shows the effects of the temperature ratio τ on $\bar{W}_i$, $\bar{W}_s$, and $\bar{W}_f$, at $E = 0.8$, $\chi = 0.5$, $e_p = e_d = 2.5$, and $\epsilon_p = \epsilon_d = 1$. It is found that both $\bar{W}_i$ and $\bar{W}_s$ are decreased, while $\bar{W}_f$ is increased, by increasing the temperature ratio. As τ is elevated to be 0.81, the magnitude of $\bar{W}_s$ vanishes to zero. This implies that the engine is not able to output and work. As the temperature ratio is further increased to be unity, even the indicated work will become zero since $\tau = 1$ means no temperature difference between the heater and the cooler.

Effects of the total dead volume ratio χ on $\bar{W}_i$, $\bar{W}_s$, and $\bar{W}_f$ are displayed in Fig. 7 for the case at $E = 0.8$, $\tau = 0.3$, $e_p = e_d = 2.5$, and $\epsilon_p = \epsilon_d = 1$. The results shown in this figure clearly reveal that the works produced by the engine are sensitive to an increase in the dead volume. It is observed that $\bar{W}_i$, $\bar{W}_s$, and $\bar{W}_f$ are all remarkably decreased by increasing the dead volume ratio χ , especially in the regime between $\chi = 0$ and 0.5. In theory, the magnitude of χ should be reduced as low as possible. However, installation of the heater, the regenerator, and the cooler is necessary in an engine in order to maintain a temperature difference (and hence, decrease the magnitude of τ). These three components and their internal dead volumes are actually inevitable, and in this regard, a study of the optimal value of the total dead volume ratio χ becomes an important issue.

Effects of e_p and e_d on the dimensionless works are conveyed in Fig. 8(a) and (b) respectively, for the case at $E = 0.8$, $\tau = 0.3$, $\chi = 0.5$,

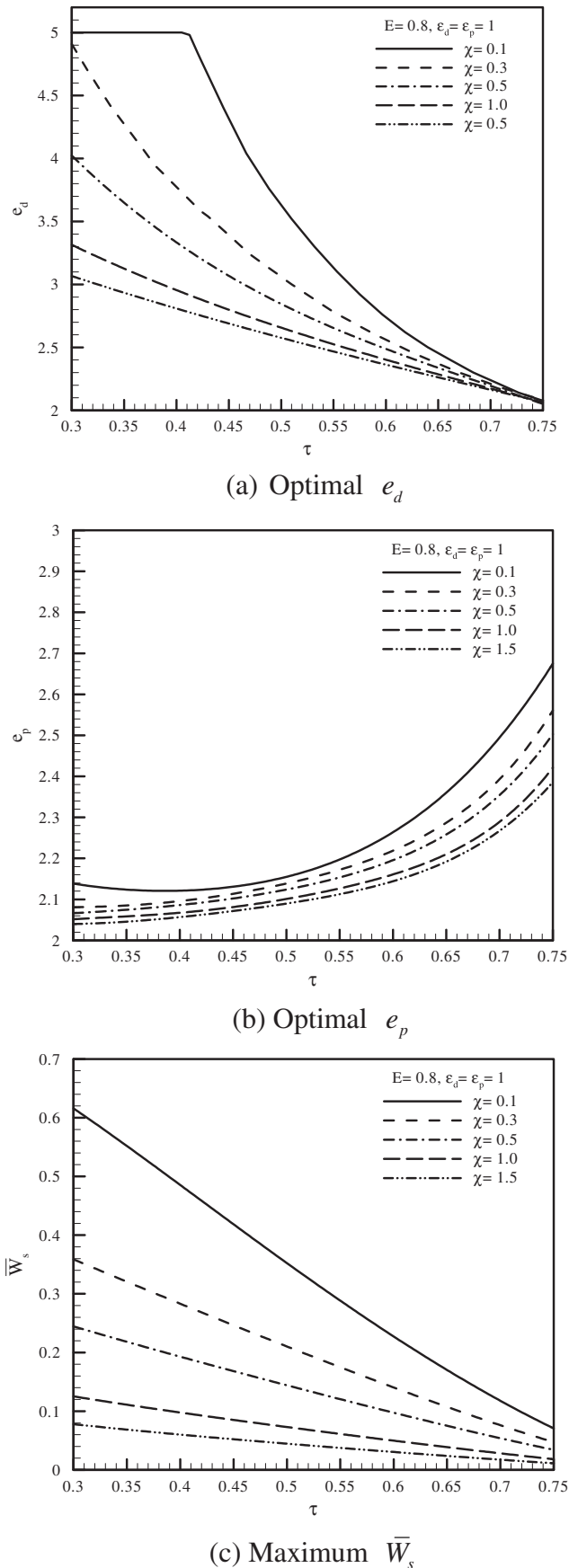


Fig. 11. Dependence of optimal $\bar{W}_s$, e_p , and e_d at $E = 0.8$ and $e_p = e_d = 1$. (a) Optimal e_d , (b) Optimal e_p , (c) Maximum $\bar{W}_s$.

and $e_p = e_d = 2.5$. In Fig. 8(a), it is found that $\bar{W}_i$ and $\bar{W}_f$ are not sensitive to a change in e_p . As e_p is elevated from 1.0 to 1.25, either $\bar{W}_i$ or $\bar{W}_f$ experiences an inappreciable increase. However, a slight decrease in $\bar{W}_s$ is observed as e_p is elevated from 1.0 to 1.25. On the contrary, Fig. 8(b) shows that both $\bar{W}_i$ and $\bar{W}_f$ are decreased by increasing e_d , and $\bar{W}_i$ is decreased faster than $\bar{W}_f$. As a result, $\bar{W}_s$ is decreased as e_d is increased. The results in Fig. 8(a) and (b) show that all the works are monotonically varied with either e_p or e_d . It appears that both e_p and e_d have to be set as small as possible to yield a high shaft work. Being aware that in accordance with the criteria of Eq. (3), e_p and e_d ought to be higher than or equal to unity. Therefore, the optimal values of e_p and e_d both are suggested to be 1.0.

Fig. 9 displays three-dimensional surface of $\bar{W}_s$ as a function of e_p and e_d for different E , at $\tau = 0.3$, $\chi = 0.5$, and $e_p = e_d = 1$. It is clearly seen in Fig. 9(a) and (b), for the cases at $E = 0.7$ and 0.8 , that a peak of $\bar{W}_s$ exists in each of the plots. In Fig. 9(a), the optimal combination of e_p and e_d is $e_p = 2.24$ and $e_d = 3.32$. However, in Fig. 9(c) for the case with a higher E , the peak of $\bar{W}_s$ is no longer visible. In this case, $\bar{W}_s$ is increased monotonically by a decrease in e_p . Therefore, under the constraint of Eq. (3), the optimal e_p is selected to be 2.0. On the other hand, at $e_p = 2$, it is observed in Fig. 9(c) that $\bar{W}_s$ increases monotonically with e_p . The selection of e_d is dependent on individual cases. However, a finite value ought to be assigned to e_p due to the limitation of links size. In this study, e_d is suggested to be 5.0.

Fig. 10 conveys the results of $\bar{W}_s$ as a function of e_p and e_d for different combinations of τ and χ , at $E = 0.8$ and $e_p = e_d = 1$. A closer look into Fig. 9(b) and Fig. 10(a) gives that as the value of τ is assigned to be 0.3 or 0.5, a peak of $\bar{W}_s$ is clearly seen. The peak is more obvious with a higher τ . Fig. 10(b) exhibits a peak of $\bar{W}_s$ at a small e_p . A comparison between Fig. 9(b) and Fig. 10(b) illustrates the influence of χ varied from 0.5 to 1.0. The points of the peaks of $\bar{W}_s$ indicate the optimal combinations of e_p and e_d under different conditions.

Next, the optimal values of e_p , e_d , and the corresponding maximum $\bar{W}_s$ are plotted as design charts that may be used to determine optimal geometric parameters and evaluate the shaft power output.

Dependence of the optimal values of e_p , e_d and their corresponding $\bar{W}_s$ on τ and χ are displayed in Fig. 11 at $E = 0.8$ and $e_d = e_p = 1.0$. Using the plots of this figure, one is able to find out the optimal values of e_p , e_d and $\bar{W}_s$ provided that the values of τ , and χ are given beforehand. For example, if $\tau = 0.5$ and $\chi = 0.5$, the optimal design of rhombic drive mechanism is: $e_p = 2.12$ and $e_d = 2.85$, and the corresponding $\bar{W}_s$ is 0.15. It is noted that for the curve of $\chi = 0.1$, e_d is fixed at 5.0 when τ is smaller than 0.42.

5. Conclusions

A dimensionless model has been developed and used to optimize the geometry of the rhombic drive mechanism with the β -type Stirling engine toward maximization of the shaft work. Displacements of the piston and the displacer with the rhombic drive mechanism and variations of volumes and pressure in the chambers of the engine are expressed in dimensionless form. Schmidt analysis is incorporated with Senft's shaft work theory to build the dimensionless thermodynamic model, in which the dimensionless shaft work $\bar{W}_s$ is treated as a function of the effectiveness of mechanism (E), the temperature ratio (τ), the total dead volume ratio (χ), and the geometric parameters (e_p , e_d , e_p , and e_d). In order to verify the thermodynamic model, the performance test of a 300-W β -type Stirling engine with rhombic drive mechanism is also conducted. It is noted that the experimental data agrees with the theoretical results with a relative error lower than 5.2%.

In addition, a parameter analysis is performed and the geometric optimization of the engine is attempted. The design charts that help determine the optimal geometric parameters of the rhombic drive mechanism are provided.

Results for a typical case with $E = 0.8$, $\chi = 0.5$, $e_p = e_d = 2.5$, and $\varepsilon_p = \varepsilon_d = 1$ show that as the temperature ratio τ is elevated to be 0.81, the magnitude of dimensionless shaft work output $\bar{W}_s$ vanishes to zero. In addition, the works produced by the engine are sensitive to an increase in the dead volume. It is observed that $\bar{W}_i$, $\bar{W}_s$, and $\bar{W}_f$ are all remarkably decreased by increasing the total dead volume ratio χ , especially in the regime between $\chi = 0$ and 0.5.

Valid ranges of the geometric parameters of the rhombic-drive mechanism are found to be $e_p - 1 > \varepsilon_p \geq 1$ and $e_d - 1 > \varepsilon_d \geq 1$. In accordance with the predictions by the model, the optimal values of ε_p and ε_d both are selected to be 1.0. On the other hand, the optimal values of e_p , e_d , and the corresponding maximum $\bar{W}_s$ are plotted as design charts that may be used to determine optimal geometric parameters and evaluate the shaft power output. For the particular case at $\tau = 0.5$ and $\chi = 0.5$, the optimal combination of e_p and e_d is: $e_p = 2.12$ and $e_d = 2.85$, and the corresponding $\bar{W}_s$ is 0.15.

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